Question and revision problem of Advanced QM

Lecture 1. Basics concepts

1) Find the complex conjugate of (a) -4 , (b) -2i , (c) 6+3i , (d) $2e^{-i/5}$.

2) Which of the following functions are normalizable over the indicated intervals?

Normalize those functions which can be normalized.

(a) $\exp(-ax^2)$ $(-\infty,\infty)$; (b) e^x $(0, \infty)$; (c) $e^{i\Phi}$ $(0,2\pi)$; (d) xe^{-3x} $(0,\infty)$

2. Determine whether each of the following functions is acceptable as a wavefunction over the indicated interval.

(a) $1/x$ $(0,\infty)$; (b) $(1-x^2)$-1 $(-1,1)$; (c) $e^{-x}\cos(x)$ $(0, \infty)$; (d) $\tan^{-1}(x)$ $(0, \infty)$

3. Which of the following operators are Hermitian

(a) i (b) * (take complex conjugate) (c) e^{ix} (d) -id/dx

(e) i^2d/dx (f) d^2dx^2 (g) id2/dx2

4. True or False

(a) Nondegenerate eigenfunctions of the same operator are orthogonal.

(b) All Hermitian operators are real.

(c) If two operators commute with a third, they will commute with each other.

(d) $d\psi/dx$ must be continuous as long as the potential, V(x), is finite.

(e) If a wavefunction is simultaneously the eigenfunction of two operators, it will also be an eigenfuncion of the product of the two operators.

5. Consider the following hypothetical PIB wavefunction: $\psi(x) = A(x-1)$ $0\leq x \leq a$

Calculate: (a) A; (b) $<x^2>$; (c) $<p>$; (d) $<p^2>$

6. Consider the functions: $\psi1 = 1$; $\psi2 = x$; $\psi3 = x^2 - 1/3$.

Show that all three functions are orthogonal over the interval [-1,1].

7. Calculate the commutator: [d/dx, d/dx + x]

8. Calculate the commutator: $[P_x,X^2]$

9. Classify the following operators as linear or nonlinear:

(a) $3x^2d^2/dx^2$; (b) $(\ )^2$ (square the function); (c) $\int(\ )dx$ (integrate the function; (d) exp ( ) (exponentiate the function)

10. Which of the following functions are eigenfunctions of d^2/dx^2 ? For those that are eigenfunctions, determine the eigenvalues.

(a) e^{2x}; (b) x^2; (c) sin(8x); (d) sin(3x) - cos(3x)

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11. Which of the following functions (defined from - ∞ to ∞) would be acceptable one dimensional wavefunctions for a bound particle. (a) exp (-ax); (b) x.exp($-bx^2$) ; (c) i.exp($-bx^2$) ; (d) sin(bx)

Lecture 2 PIB

If m = $1x10^{-3}$ kg A = 0.10 m and h = $6.63x10^{-34}$ J-s

Q1) For a particle in a box with Wavefunctions and Energy:

$$\psi_n = A\sin(\alpha x) = A\sin\left(\frac{n\pi}{a}x\right) \qquad E_n = \frac{n^2h^2}{8ma^2}$$

I. Show that the wavefunction is an eigenfunction of the Hamiltonian operator.
II. Normalize the wavefunction and find A
III. Calculate the following quantities: $\langle x \rangle$ $\langle x^2 \rangle$ $\langle p \rangle$

$\langle p^2 \rangle$ σ_x^2 σ_p^2 $\langle KE \rangle$ $\langle PE \rangle$ $\Delta x \, \Delta P$

IV. Find the number and position of nodes for n= 1, 2,3 4

V. Find the probability of finding the particle in

I. $0.24L \le x \le 0.26L$

II. $0. \le x \le 0.25L$

Q1 Consider an electron in a 1 Angstrom box. Calculate

(a) The Zero Point Energy (i.e. minimum energy)

(b) The minimum speed of the electron

Q2) Consider a 1 gram particle in a 10 cm box. Calculate

(a) The Zero Point Energy (i.e. minimum energy)

(b) the minimum speed of the particle

Q3) Calculate the probability of finding a particle with n=1 in the region of the box between 0 and a/4

Q4) Show that the two lowest wavefunctions $\psi 1$ and $\psi 2$ of the PIB are orthogonal:

Q5) Show that the two lowest wavefunctions $\psi 1$ and $\psi 1$ of the PIB are orthonormal